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Casimir force computations in non-trivial geometries using *Mathematica*

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Abstract

Dispersion forces between atoms and between macroscopic surfaces play remarkably important roles in such diverse systems as cold atomic traps, the remarkable climbing abilities of the *Gecko* lizard, and in micro-electro-mechanical systems (MEMS) engineering. As first anticipated by Feynman as early as 1959, van der Waals interactions pose critical limitations to MEMS performance as they are the fundamental cause of stiction, that is, the permanent latching of microparts during fabrication or operation. Motivated by the ongoing downscaling of MEMS designs to regimes in which van der Waals forces are expected to diverge and to dominate the dynamics of the system, attempts have been made to understand stiction in cases other than the very few archetypal geometries for which analytical solutions are known to exist. Additionally, as pointed out by this author, dispersion forces represent not only a limitation but actually offer a unique engineering opportunity for contactless actuation on the nanoscale by means of strategies that manipulate the quantum vacuum. These strategies are based on the specific properties that are known to drastically affect dispersion forces, such as, for example, the conductivity or the magnetic permeability of the interacting boundaries and the topology of the problem. It has been known for many years that, for instance, van der Waals forces can become repulsive under specific circumstances, although this fascinating phenomenon has not yet been observed experimentally.

Despite the pressing need to include dispersion forces in the design of micro- and nano-device performance analysis, the goal of even formulating the numerical problem has remained elusive. The absence of an accurate computational dispersion force algorithm was dramatically illustrated by recent a experiment on the Casimir force between a sphere and a plate. This effort, aimed at investigating the breakdown of the typical perturbative treatment based on the proximity force theorem, revealed that no existing theory made predictions in agreement with the experimental data.

In the last year, however, this unacceptable situation has suddenly changed with the introduction of methods based on the computation of the Green function for imaginary times, based on numerical approaches quite familiar from classical cavity electrodynamics. It appears now, for the first time, that a reliable framework exists for the computation of Casimir forces between objects with arbitrary geometries by means of finite difference methods.

Since the appearance of two groundbreaking articles in this field, the present author has investigated the application of *Mathematica* to this problem with the intention to both make the treatment as transparent as possible and to make progress towards more complex cases. In this presentation, we shall briefly introduce the relevant mathematical problem and its challenges by means of *Mathematica*'s graphic capabilities. We shall then analyze the use of *Mathematica* to carry out the extremely intensive algebraic computations to obtain the difference equations that describe the problem. This approach, which is derived from recent developments in the analysis of waveguides of arbitrary cross section, has already allowed the author to produce the first example of accurate Casimir force computation with sub-pixel accuracy in one-dimensional systems. We shall proceed by illustrating the use of an enhanced approach of this type to treat two-dimensional, axial geometries, which lead to extremely awkward matrix computations to produce the needed difference equations. Finally, we shall analyze the use of *Mathematica* to carry out the imaginary frequency integrations needed to obtain the value of the Casimir force. The presentation shall conclude with illustrations of applications to various systems of engineering and fundamental physics interest.